Journal of Engineering Physics and Thermophysics, Vol. 79, No. 4, 2006

HEAT TRANSFER FROM TUBES FILLED WITH A GRANULAR LAYER WITH ACCOUNT FOR ITS THERMAL ANISOTROPY

M. V. Vinogradova and Yu. S. Teplitskii UDC 532.546

The heat transfer from a tube filled with an infiltrated granular layer has been simulated with account for the parallel thermal conductivity of this layer at the first-kind and second-kind boundary conditions at the outer surface of the tube with the use of the Danckwerts conditions. Analytical dependences determining the influence of the parallel thermal conductivity of the granular layer on the average at its cross section as well as on the heat flow and the heat transfer in it have been obtained. Dependences for calculating the active length of a heat exchanger are presented.

The phenomena of convective heat transfer in porous media and granular layers, including the heat exchange in phase transformations, are of great interest because of the large number of applications. Among them are the removal of heat from the macrowastes of a radioactive combustible in the case of a nuclear reactor accident, the heat and mass transfer in the wastes of a nuclear fuel buried in deep natural geological spaces, the underground transfer of contaminants, the development of sunlight detectors, the development of chemical-engineering apparatus in which reactors with a high-density packed granular layer are used and optimization of processes occurring in them, and the design of nuclear reactors with spherical bodies. Of great practical interest is the study of the heat and mass transfer in the process of combustion of a solid organic fuel in an immovable layer. An understanding of the main mechanisms of the transfer processes occurring in porous and granular media is necessary for the development of regenerative heat exchangers, high-efficiency thermal insulation for buildings and refrigerators, and high-efficiency methods of drying.

Since the processes pf heat transfer in infiltrated granular media proceed in complex-geometry channels, many additional factors (as compared to single-phase media) influence their intensity and character. These factors are first of all the diameter of the grains, their shape and thermophysical properties, and the character of packing of them (especially near the macrosurfaces) determining the near-wall thermal resistance. An important property of an infiltrated dispersive medium is the anisotropy of its effective diffusion and thermal conductivity, determining the transfer processes in it. The velocity distribution of a heat-transfer agent flow in the cross section of a granular medium, which substantially influences the heat-transfer processes in this medium, is also determined by its geometry: it differs from the velocity distribution of a one-phase flow.

A large number of works are devoted to investigating the influence of different factors on the heat transfer between a granular medium and the surface of a tube (see, e.g., [1–5]). However, the influence of the thermal anisotropy of an infiltrated granular layer on its thermal state and the heat transfer between it and the surface of a tube was practically not investigated. In this connection, the aim of the present work is to investigate the total influence of the effective parallel and cross-field thermal conductivities of a granular layer on its thermal characteristics at different boundary conditions.

First-Kind Boundary Conditions. For a steady-state heat exchange between a granular layer and a tube, the heat-conduction equation (two zone one-temperature model) has the form

$$c_{\mathrm{f}}\,\rho_{\mathrm{f}} u \frac{\partial T}{\partial x} = \lambda_r \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \right) + \lambda_x \frac{\partial^2 T}{\partial x^2} \,. \tag{1}$$

A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, 15 P. Brovka Str., Minsk, 220072, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 79, No. 4, pp. 153–164, July–August, 2006. Original article submitted July 8, 2005.

 1062-0125/06/7904-0788

at the boundary conditions

$$r = 0\,,\ \ \frac{\partial T}{\partial r} = 0\,;\ \ r = R - l\,,\ \ -\lambda_r \frac{\partial T}{\partial r} = K\,(T - T_0)\,;$$

$$x = 0\,,\ \ c_{\mathrm{f}}\,\rho_{\mathrm{f}} u T^{\mathrm{in}} = c_{\mathrm{f}}\,\rho_{\mathrm{f}} u T - \lambda_x \frac{\partial T}{\partial x}\,;\ \ x = L\,,\ \ \frac{\partial T}{\partial x} = 0\,, \tag{2}$$

where $K = (1/\alpha_{\mathrm{w}} + \delta_{\mathrm{m}}/\lambda_{\mathrm{m}})^{-1}$ is the heat-transfer coefficient accounting for the thermal resistance of the near-wall zone $1/\alpha_{\mathrm{w}}$ and the wall of the tube $\delta_{\mathrm{m}}/\lambda_{\mathrm{m}}$. The near-wall heat-transfer coefficient is determined by the dependence [5]

$$\alpha_{\mathrm{w}} = \frac{10}{d}\,(A\lambda_{\mathrm{f}} + 0.0061 c_{\mathrm{f}}\,\rho_{\mathrm{f}} u d)\,, \tag{3}$$

where A = 1.6 for heat-conducting particles and A = 1 for thermally nonconductive particles. The cross-field thermal conductivity of the granular layer is determined from the formula [6]

$$\lambda_r = \lambda_{\mathrm{s}}^0 + 0.1 c_{\mathrm{f}}\,\rho_{\mathrm{f}} u d\,. \tag{4}$$

At a filtration rate u = 0, the thermal conductivity of the granular layer is calculated by the dependence [6]

$$\lambda_{\mathrm{s}}^0 = \lambda_{\mathrm{f}} + \frac{\lambda_{\mathrm{f}}\,(1 - \varepsilon)\,(1 - \lambda_{\mathrm{f}}/\lambda_{\mathrm{s}})}{\lambda_{\mathrm{f}}/\lambda_{\mathrm{s}} + 0.28\,\exp\,(0.63(\lambda_{\mathrm{s}}/\lambda_{\mathrm{f}})^{0.18})}\,. \tag{5}$$

The parallel thermal conductivity of the granular layer is determined as

$$\lambda_x = 0.66\mathrm{Re}_{\mathrm{e}}^{0.3}\lambda_r\,. \tag{6}$$

This expression is true at Re_{e} = 10–4000. Formula (b) was obtained by us with the use of experimental data of [1, Fig. IV.7]. As is seen, an infiltrated granular layer is an anisotropic medium with substantially different coefficients λ_r and λ_x. Note that, at x = 0 and x = L, the boundary conditions represent the known Danckwerts conditions [7].

Let us write system (1), (2) in the dimensionless form:

$$\frac{\partial \theta}{\partial x'} = \frac{\partial^2 \theta}{\partial\,(r')^2} + \frac{1}{r'}\frac{\partial \theta}{\partial r'} + \frac{1}{\mathrm{Pe}^2}\frac{\partial^2 \theta}{\partial\,(x')^2}\,,$$

$$r' = 0\,,\ \ \frac{\partial \theta}{\partial r'} = 0\,;\ \ r' = 1 - \frac{l}{R}\,,\ \ \mathrm{Bi}\,\theta + \frac{\partial \theta}{\partial r'} = 0\,; \tag{7}$$

$$x' = 0\,,\ \ \theta = 1 + \frac{1}{\mathrm{Pe}^2}\frac{\partial \theta}{\partial x'}\,;\ \ x' = L'\,,\ \ \frac{\partial \theta}{\partial x'} = 0\,.$$

The problem will be solved with the use of the Hankel finite integral transform [8] with respect to the variables r':

$$\tilde{\theta}\,(\mu_n, x') = \int_0^1 r'\theta\,(r', x')\,J_0\,(\mu_n r')\,dr'\,, \tag{8}$$

where μ_n are roots of the characteristic equation $\dfrac{J_0(\mu_n)}{J_1(\mu_n)} = \dfrac{\mu_n}{\mathrm{Bi}}$. Let us apply transform (8) to system (7). Using the relation [8]

$$\int_0^1 \left(\frac{\partial^2 \theta}{\partial (r')^2} + \frac{1}{r'} \frac{\partial \theta}{\partial r'} \right) r' J_0 (r' \mu_n)\, dr' = - \mu_n^2 \tilde{\theta} \tag{9}$$

we will obtain a differential equation in the region of representations

$$\frac{d^2 \tilde{\theta}}{d (x')^2} - \mathrm{Pe}^2 \frac{d\tilde{\theta}}{dx'} - \mathrm{Pe}^2\, \mu_n^2 \tilde{\theta} = 0 \tag{10}$$

and the boundary conditions

$$x' = 0\,, \quad \tilde{\theta} = \frac{J_1 (\mu_n)}{\mu_n} + \frac{1}{\mathrm{Pe}^2} \frac{d\tilde{\theta}}{dx'}\,; \quad x' = L'\,, \quad \frac{d\tilde{\theta}}{dx'} = 0\,. \tag{11}$$

Solution of (10), (11) gives

$$\tilde{\theta} = \frac{J_1 (\mu_n)\, \mathrm{Pe}^2 \left(s_{2n} \exp (s_{2n} L' + s_{1n} x') - s_{1n} \exp (s_{1n} L' + s_{2n} x')\right)}{\mu_n \left(s_{2n}^2 \exp (s_{2n} L') - s_{1n}^2 \exp (s_{1n} L')\right)}\,, \tag{12}$$

where

$$s_{1n} = \frac{\mathrm{Pe}^2}{2} - \frac{\mathrm{Pe}^2}{2} \sqrt{1 + 4 \frac{\mu_n^2}{\mathrm{Pe}^2}}\,; \quad s_{2n} = \frac{\mathrm{Pe}^2}{2} + \frac{\mathrm{Pe}^2}{2} \sqrt{1 + 4 \frac{\mu_n^2}{\mathrm{Pe}^2}}\,.$$

The change from the representation of the function $\tilde{\theta}(\mu_n, x')$ to its original $\theta(r', x')$ is performed by the formula [8]

$$\theta (r', x') = \sum_{n=1}^{\infty} \frac{2\mu_n^2}{\mathrm{Bi}^2 + \mu_n^2} \frac{J_0 (\mu_n r')}{J_0^2 (\mu_n)} \tilde{\theta} (\mu_n, x')\,. \tag{13}$$

The desired solution has the form

$$\theta = \sum_{n=1}^{\infty} \frac{2\mathrm{Bi}}{\mathrm{Bi}^2 + \mu_n^2} \frac{J_0 (\mu_n r')}{J_0 (\mu_n)} \left(C_{1n} \exp (s_{1n} x') + C_{2n} \exp (s_{2n} x')\right)\,, \tag{14}$$

where

$$C_{1n} = \frac{\mathrm{Pe}^2\, s_{2n} \exp (s_{2n} L')}{s_{2n}^2 \exp (s_{2n} L') - s_{1n}^2 \exp (s_{1n} L')}\,;$$

$$C_{2n} = \frac{\mathrm{Pe}^2\, s_{1n} \exp (s_{1n} L')}{s_{1n}^2 \exp (s_{1n} L') - s_{2n}^2 \exp (s_{2n} L')}\,.$$

The dimensionless temperature averaged over the cross section of the tube is determined as

$$\langle \theta \rangle = \frac{\langle T \rangle - T_0}{T^{\mathrm{in}} - T_0} = \sum_{n=1}^{\infty} \frac{4\mathrm{Bi}^2}{(\mathrm{Bi}^2 + \mu_n^2)\, \mu_n^2} \left(C_{1n} \exp (s_{1n} x') + C_{2n} \exp (s_{2n} x')\right)\,. \tag{15}$$

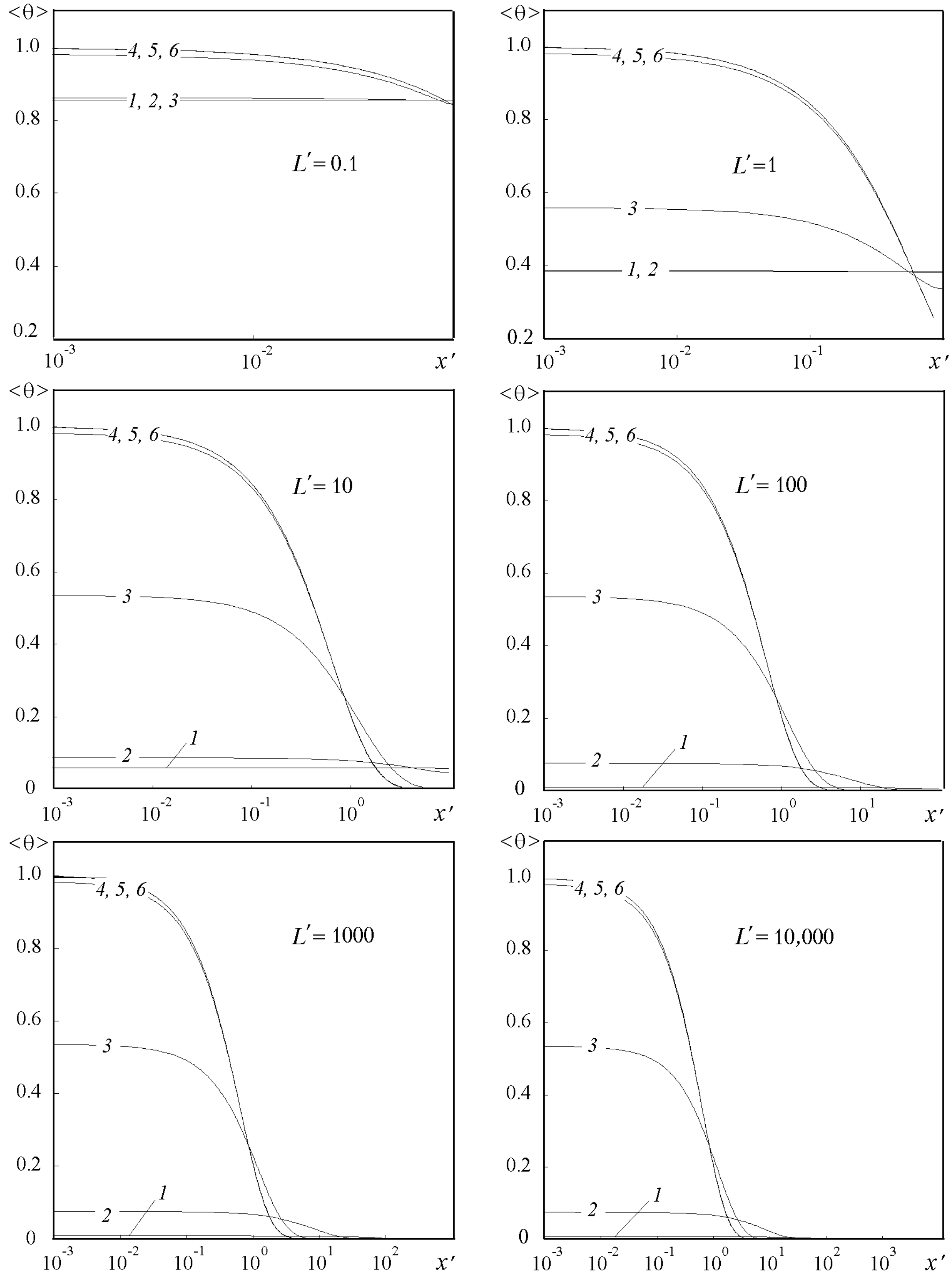

Fig. 1. Dependence of the average dimensionless temperature on x' (calculation by (15)) at Bi = 1: Pe = 0.01 (1), 0.1 (2), 1 (3), 10 (4), 100 (5), and ∞ (6).

The influence of the parallel thermal conductivity on the average dimensionless temperature is illustrated in Fig. 1. It is seen that, when Pe decreases, the heat-transfer agent is completely cooled at larger values of x', which can be explained by the influence of the longitudinal dispersive heat flow. For calculating the average dimensionless temperature at $x' = L'$ on the basis of approximation (15), we derived the following formulas (providing a roof-mean-square deviation of 5%):

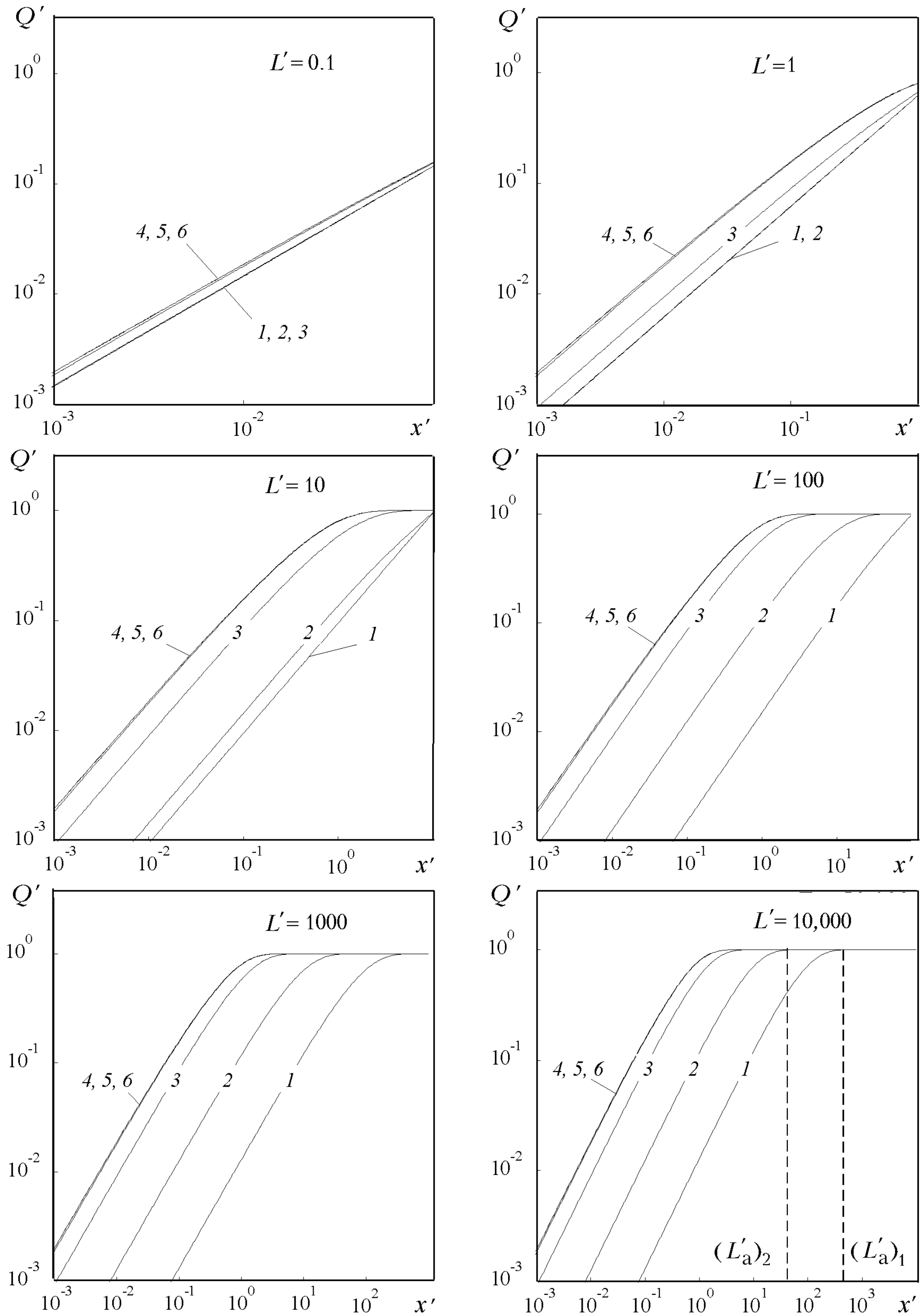

Fig. 2. Dependence of the dimensionless heat flow on x' (calculation by (17)) at Bi = 1. Designations 1–6 are identical to those in Fig. 1.

$$\langle \theta \rangle = \frac{\exp\left(\dfrac{-\mathrm{Bi}\,L'}{\mathrm{Pe}^{-1.64}\mathrm{Bi} + 0.79\mathrm{Pe}^{-0.054}}\right)}{1 + 0.96\mathrm{Bi}\mathrm{Pe}^{-0.17}L'}, \quad \mathrm{Bi} = 0.01 \ldots 1\,;$$

$$\langle\theta\rangle = \frac{\exp\left(\dfrac{-\mathrm{Bi}\,L'}{0.35\mathrm{Pe}^{0.04}\mathrm{Bi} + 3.68\mathrm{Pe}^{-1}}\right)}{1 + 1.59\mathrm{Bi}^{0.49}\mathrm{Pe}^{-0.024}\,L'}\,, \quad \mathrm{Bi} = 1 \ldots 10\,; \tag{16}$$

$$\langle\theta\rangle = \frac{\exp\left(\dfrac{-\mathrm{Bi}\,L'}{0.4\mathrm{Pe}^{-0.1}\mathrm{Bi} + 21.54\mathrm{Pe}^{-1.73}}\right)}{1 + 6.44\mathrm{Bi}^{-0.0035}\mathrm{Pe}^{-0.052}\,L'}\,, \quad \mathrm{Bi} = 10 \ldots 100\,.$$

The dimensionless heat flow removed from the region of length x of the heat exchanger is calculated by the formula

$$Q' = \frac{Q}{\pi R^2 c_\mathrm{f}\,\rho_\mathrm{f} u\,(T^\mathrm{in} - T_0)} = \sum_{n=1}^{\infty} \frac{4\mathrm{Bi}^2}{\mathrm{Bi}^2 + \mu_n^2}\left(\frac{C_{1n}\exp(s_{1n}x')}{s_{1n}} + \frac{C_{2n}\exp(s_{2n}x')}{s_{2n}}\right) -$$

$$-\sum_{n=1}^{\infty} \frac{4\mathrm{Bi}^2}{\mathrm{Bi}^2 + \mu_n^2}\left(\frac{C_{1n}}{s_{1n}} + \frac{C_{2n}}{s_{2n}}\right). \tag{17}$$

Figure 2 shows the dependence of the value of Q' on x'. It is seen that, in a heat exchanger having a fairly large length, at certain values of L_a' (see Fig. 2 for $L' = 10{,}000$) Q' ceases to depend on x' and reaches a constant value corresponding to the limiting value of the total flow ($Q' = 1$). The quantity L_a' is the active length of the heat exchanger, i.e., the length along which it exchanges a marked amount of heat with the environment. This length is determined from the condition

$$1 - Q' \le 0.05\,. \tag{18}$$

Tanalysis of the dependences $Q' = f(x', \mathrm{Pe}, \mathrm{Bi}, L')$, The following formulas were obtained for calculating the active length of a heat exchanger on the basis of numerical

$$L_\mathrm{a} = \frac{R^2 c_\mathrm{f}\,\rho_\mathrm{f} u}{\lambda_r}\,(1.45\mathrm{Bi}^{-1} + 2.14\mathrm{Bi}^{-0.45}\mathrm{Pe}^{-1})\,, \quad \mathrm{Bi} = 0.01 \ldots 1\,;$$

$$L_\mathrm{a} = \frac{R^2 c_\mathrm{f}\,\rho_\mathrm{f} u}{\lambda_r}\,(1.04\mathrm{Bi}^{-0.24} + 1.7\mathrm{Bi}^{-0.13}\mathrm{Pe}^{-1})\,, \quad \mathrm{Bi} = 1 \ldots 100\,. \tag{19}$$

Approximationly (17) gives a relation for determining (with a roof-square deviation of 5%) the value of the total heat flow removed from the heat exchanger (for $x' = L'$):

$$Q' = \frac{2\mathrm{Bi}\,L'}{\mathrm{Bi}\,(2L' + 0.15\mathrm{Pe}^{-0.11}) + 0.85\mathrm{Pe}^{-0.03}}\,, \quad L' < L_\mathrm{a}'\,; \tag{20}$$

$$Q_\infty' = 1\,, \quad L' \ge L_\mathrm{a}'\,. \tag{21}$$

Note that (21) follows from (20) at $L' \to \infty$ and from the condition of thermal balance of the system as a whole

$$Q_\infty = \pi R^2 c_\mathrm{f}\,\rho_\mathrm{f} u\,(T^\mathrm{in} - T_0)\,. \tag{22}$$

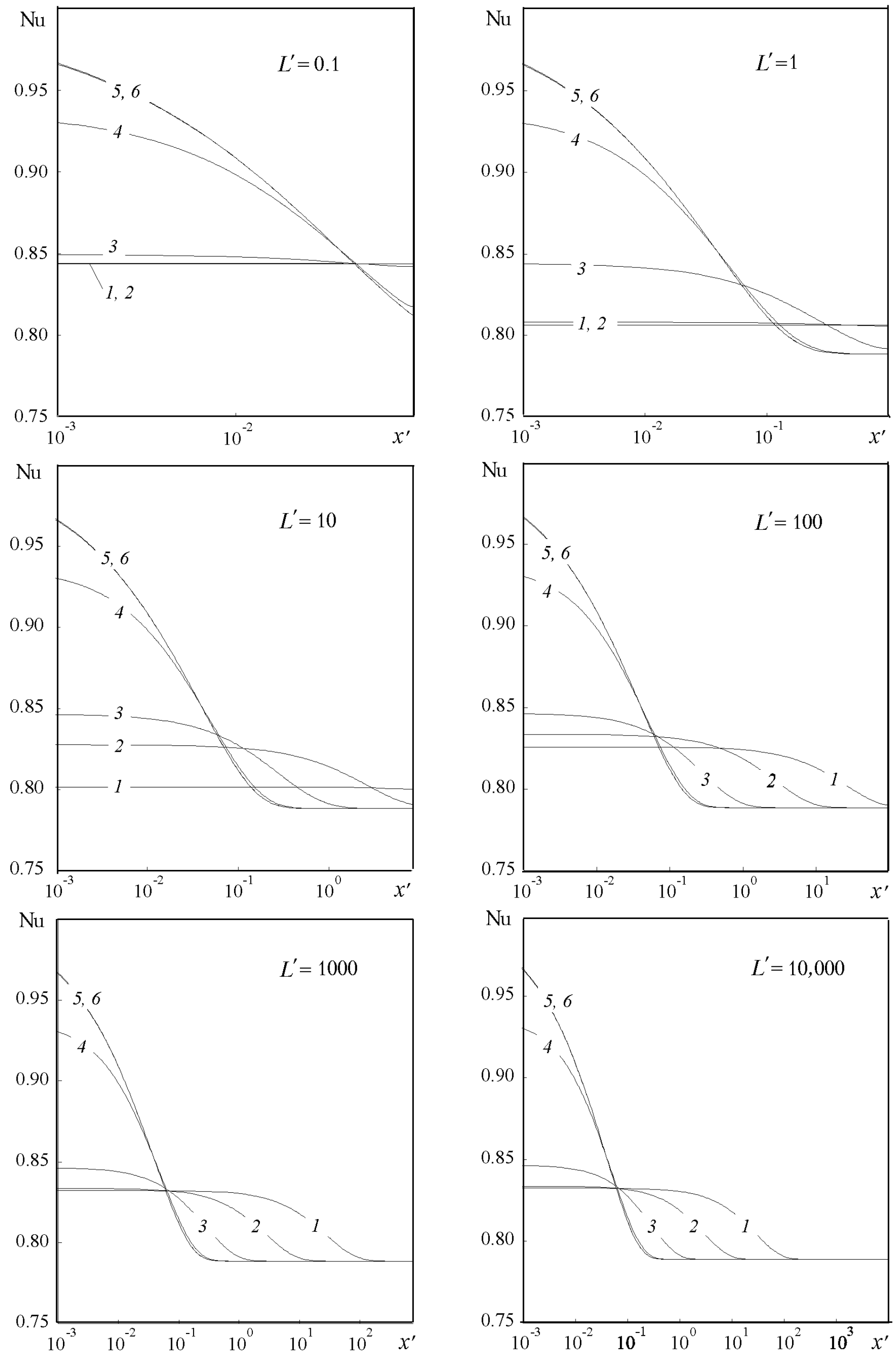

Fig. 3. Dependence of the dimensionless heat-transfer coefficient on x' at the first-kind boundary conditions (calculation by (26)) at Bi = 1. Designations 1–6 are identical to those in Fig. 1.

Relations (16) and (20) can be used for selecting the geometric characteristics of a heat exchanger (length and radius of the tube) filled with a granular layer at definite values of the flow rate of a heat-transfer agent, of the temperature of the heat-transfer agent at the input and output of the heat exchanger, of the ambient temperature, and of the thermophysical characteristics of the medium, the material of the particles, and the tube walls. The calculation is performed by the standard method of successive approximations at a definite value of Bi with the use of one of the relations (16). After a calculation cycle is completed, the range into which Bi falls is verified and, if necessary, another computational relation (16) is selected, and so on.

For the heat-transfer coefficient determined by the relation

$$K_{\Sigma} = K \frac{T\big|_{r'=1-l/R} - T_0}{\langle T \rangle - T_0}, \tag{23}$$

from (14), at $l << R$, and from the expansion

$$\int_0^1 J_0 (\mu_n r') \, r' dr' = \frac{1}{\mu_n} J_1 (\mu_n) \tag{24}$$

we obtain the dependence

$$K_{\Sigma} = K \frac{\sum\limits_{n=1}^{\infty} \frac{\text{Bi}^2}{(\text{Bi}^2 + \mu_n^2)} (C_{1n} \exp (s_{1n} x') + C_{2n} \exp (s_{2n} x'))}{2 \sum\limits_{n=1}^{\infty} \frac{\text{Bi}^2}{(\text{Bi}^2 + \mu_n^2) \, \mu_n^2} (C_{1n} \exp (s_{1n} x') + C_{2n} \exp (s_{2n} x'))}, \tag{25}$$

which is written in the dimensionless form as

$$\text{Nu} = \frac{\sum\limits_{n=1}^{\infty} \frac{1}{(\text{Bi}^2 + \mu_n^2)} (C_{1n} \exp (s_{1n} x') + C_{2n} \exp (s_{2n} x'))}{2 \sum\limits_{n=1}^{\infty} \frac{1}{(\text{Bi}^2 + \mu_n^2) \, \mu_n^2} (C_{1n} \exp (s_{1n} x') + C_{2n} \exp (s_{2n} x'))} . \tag{26}$$

To estimate the influence of the parallel thermal conductivity on the heat-transfer coefficient, we constructed the graphs Nu = $f(x')$ at different values of Bi, Pe, and L'. Such dependences, constructed for Bi = 1, are shown in Fig. 3. As is seen, the stationary value of Nu = Nu* is independent of Pe. The length of the thermal-stabilization region increases with decrease in Pe.

In the case where Pe $\to \infty$ (parallel thermal conductivity is absent), the value of Nu is determined by a formula following from (26):

$$\text{Nu} = \frac{\sum\limits_{n=1}^{\infty} \frac{1}{(\text{Bi}^2 + \mu_n^2)} \exp (- \mu_n^2 x')}{2 \sum\limits_{n=1}^{\infty} \frac{1}{(\text{Bi}^2 + \mu_n^2) \, \mu_n^2} \exp (- \mu_n^2 x')} . \tag{27}$$

It should be noted that expression (27) was also obtained in [5]. The series in (27) converge quickly; therefore, at $x' > 0.5$ (this corresponds to $x > 0.5 c_{\text{f}} \rho_{\text{f}} u R^2 / \lambda_r$) we will restrict our consideration to their first terms and obtain the following expressions for the stationary value of Nu:

$$\text{Nu}^* = \frac{\mu_1^2}{2} = \frac{1}{\frac{1}{\text{Bi}} + \frac{1}{2.8915}} . \tag{28}$$

Expression (28) was derived with the use of the relation [5]

$$\mu_1 = \sqrt{\frac{2\mathrm{Bi}}{1 + \mathrm{Bi}/2.8915}} \,. \tag{29}$$

At larger x', we can restrict our consideration to the first term in the series of formula (26); in this case, formula (26) will be also reduced to (28).

Second-Kind Boundary Conditions. The boundary conditions for the heat-conduction equation (1) for a heating granular layer are as follows:

$$r = 0\,, \;\; \frac{\partial T}{\partial r} = 0\,; \;\; r = R - l\,, \;\; \lambda_r \frac{\partial T}{\partial r} = q = \mathrm{const}\,;$$
$$x = 0\,, \;\; c_{\mathrm{f}}\,\rho_{\mathrm{f}} u T^{\mathrm{in}} = c_{\mathrm{f}}\,\rho_{\mathrm{f}} u T - \lambda_x \frac{\partial T}{\partial x}\,; \;\; x = L\,, \;\; \frac{\partial T}{\partial x} = 0\,. \tag{30}$$

Let us write (1) and (30) in the dimensionless form

$$\frac{\partial \theta}{\partial x'} = \frac{\partial^2 \theta}{\partial (r')^2} + \frac{1}{r'}\frac{\partial \theta}{\partial r'} + \frac{1}{\mathrm{Pe}^2}\frac{\partial^2 \theta}{\partial (x')^2}\,,$$

$$r' = 0\,, \;\; \frac{\partial \theta}{\partial r'} = 0\,; \;\; r' = 1 - \frac{l}{R}\,, \;\; \frac{\partial \theta}{\partial r'} = \frac{q}{\lambda_r}\frac{R}{T^{\mathrm{in}}} = \overline{Q}\,; \tag{31}$$

$$x' = 0\,, \;\; \frac{1}{\mathrm{Pe}^2}\frac{\partial \theta}{\partial x'} = \theta\,; \;\; x' = L'\,, \;\; \frac{\partial \theta}{\partial x'} = 0\,.$$

The problem will be solved using the Hankel finite integral transform with respect to the variable r' (8) on the assumption that μ_n are roots of the characteristic equation $J_1(\mu_n) = 0$. Let us apply transform (8) to Eq. (31). Using the relation [8]

$$\int_0^1 \left(\frac{\partial^2 \theta}{\partial (r')^2} + \frac{1}{r'}\frac{\partial \theta}{\partial r'} \right) r' J_0 (r'\mu_n)\, dr' = \overline{Q} J_0 (\mu_n) - \mu_n^2 \tilde{\theta} \tag{32}$$

we will obtain the differential equation in the region of representations

$$\frac{d^2 \tilde{\theta}}{d (x')^2} - \mathrm{Pe}^2 \frac{d\tilde{\theta}}{dx'} - \mathrm{Pe}^2 \mu_n^2 \tilde{\theta} = - \mathrm{Pe}^2 J_0 (\mu_n)\, \overline{Q} \tag{33}$$

and the boundary conditions

$$x' = 0\,, \;\; \frac{1}{\mathrm{Pe}^2}\frac{d\tilde{\theta}}{dx'} = \tilde{\theta}\,; \;\; x' = L'\,, \;\; \frac{d\tilde{\theta}}{dx'} = 0\,. \tag{34}$$

Solution of (33), (34) gives

$$\tilde{\theta} = \frac{J_0 (\mu_n)\, \overline{Q}}{\mu_n^2} \left(\frac{\mathrm{Pe}^2 \left(s_{1n} \exp (s_{1n} L' + s_{2n} x') - s_{2n} \exp (s_{2n} L' + s_{1n} x') \right)}{s_{2n}^2 \exp (s_{2n} L') - s_{1n}^2 \exp (s_{1n} L')} + 1 \right). \tag{35}$$

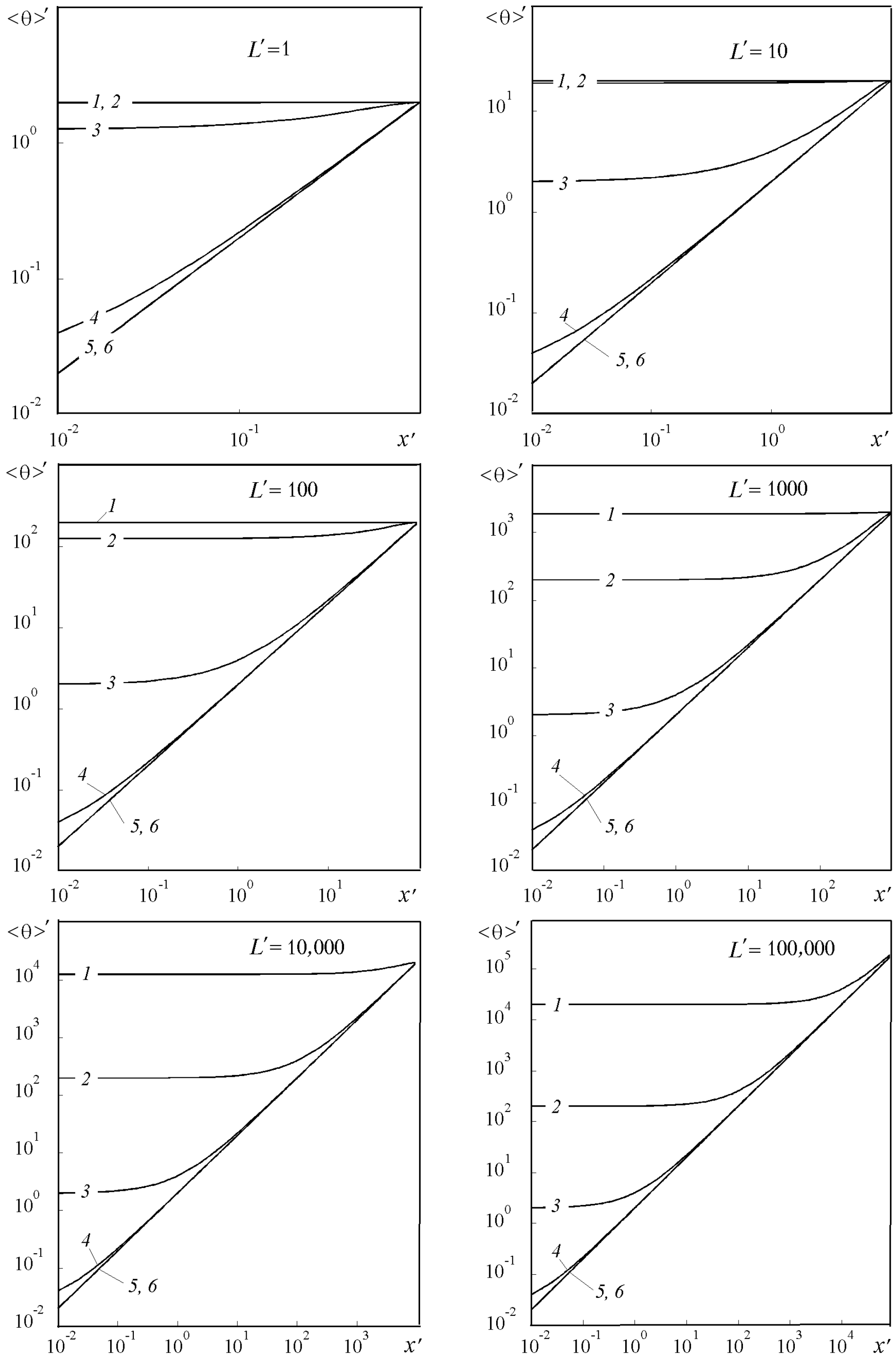

Fig. 4. Dependence of the average reduced dimensional temperature on x' (calculation by (38)). Designations 1–6 are identical to those in Fig. 1.

The change from the representation of the function $\tilde{\theta}(\mu_n, x')$ to its original $\theta(r', x')$ is performed by the formula [8]

$$\theta(r', x') = 2\tilde{\theta}(0, x') + 2\sum_{n=1}^{\infty} \tilde{\theta}(\mu_n, x') \frac{J_0(\mu_n r')}{J_0^2(\mu_n)} . \tag{36}$$

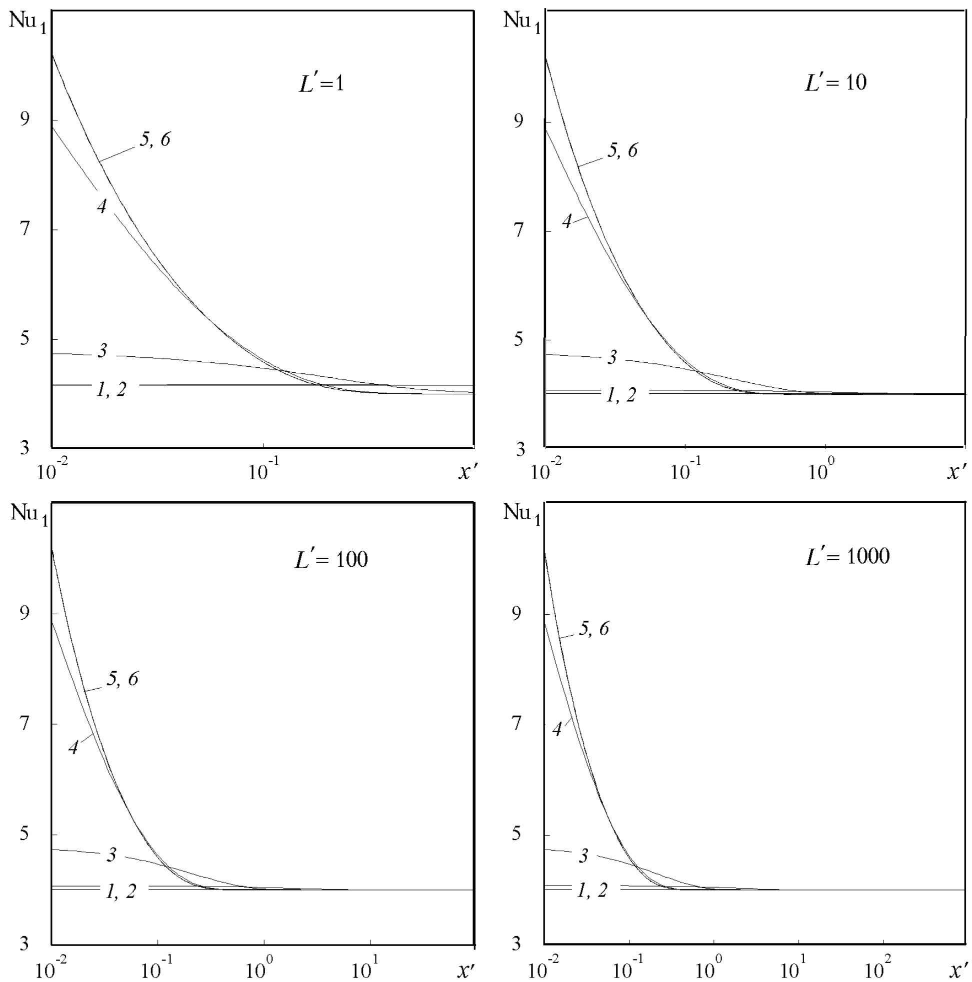

Fig. 5. Dependence of the dimensionless heat-transfer coefficient on x' at the second-kind boundary conditions (calculation by (41)). Designations 1–6 are identical to those in Fig. 1.

As a result, we obtain

$$\theta = \overline{Q}\left(2x' - 2\sum_{n=1}^{\infty}\frac{J_0(\mu_n r')}{J_0(\mu_n)\,\mu_n^2}\left(C_{1n}\exp(s_{1n}x') + C_{2n}\exp(s_{2n}x')\right) - \frac{1}{4} + \frac{(r')^2}{2}\right) + \frac{2\overline{Q}}{\mathrm{Pe}^2}\left(1 - \exp\left(-\mathrm{Pe}^2(L' - x')\right)\right), \qquad (37)$$

where the quantities C_{1n} and C_{2n} are identical to those in (14).

The dimensionless temperature averaged over the cross section of the tube is determined by the formula

$$\langle\theta\rangle = \frac{\langle T\rangle - T^{\mathrm{in}}}{T^{\mathrm{in}}} = \overline{Q}\left\{2x' + \frac{2}{\mathrm{Pe}^2}\left[1 - \exp\left(-\mathrm{Pe}^2(L' - x')\right)\right]\right\}, \qquad (38)$$

obtained on the basis of (37). Figure 4 shows the dependences of the reduced temperature $\langle\theta\rangle' = \langle\theta\rangle/\overline{Q}$ on x' at different values of Pe and L'. It should be noted that, at small values of Pe (large coefficients λ_x) there is a region, in

TABLE 1. Values of the Heat-Transfer Coefficient at Different Reynolds Numbers

Re_D	α^*	α'
500	6.17	0.72
1000	6.81	0.72
2000	8.10	0.72
10,000	18.08	4.71
20,000	30.25	8.20
50,000	66.27	17.07

which the temperature changes insignificantly. This phenomenon is due to the influence of the dispersive heat backflow $\left(-\lambda_x \frac{\partial T}{\partial x}\right)$ directed to the input region of the tube.

For the heat-transfer coefficient, determined by the relation

$$\alpha_1 = \frac{q}{T\big|_{r'=1-l/R} - \langle T \rangle}, \tag{39}$$

we obtain, using the condition $l \ll R$ and expression (37), the dependence

$$\alpha_1 = \frac{1}{\frac{R}{\lambda_r}\left(\frac{1}{4} - 2\sum_{n=1}^{\infty}\frac{1}{\mu_n^2}\left(C_{1n}\exp\left(s_{1n}x'\right) + C_{2n}\exp\left(s_{2n}x'\right)\right)\right)}, \tag{40}$$

and write it in the dimensionless form

$$\mathrm{Nu}_1 = \frac{1}{\frac{1}{4} - 2\sum_{n=1}^{\infty}\frac{1}{\mu_n^2}\left(C_{1n}\exp\left(s_{1n}x'\right) + C_{2n}\exp\left(s_{2n}x'\right)\right)}. \tag{41}$$

If the influence of the parallel thermal conductivity is disregarded ($\mathrm{Pe} \to \infty$), (41) takes the form

$$\mathrm{Nu}_1 = \frac{1}{\frac{1}{4} - 2\sum_{n=1}^{\infty}\frac{\exp\left(-\mu_n^2 x'\right)}{\mu_n^2}}. \tag{42}$$

From (41) and (42) we obtain the following relation for calculating the stationary value of the heat-transfer coefficient at $x' \to \infty$:

$$\mathrm{Nu}_1^* = 4\,. \tag{43}$$

Figure 5 presents the dependences $\mathrm{Nu}_1 = f(x')$ obtained for different values of Pe and L'. It is seen that, when Pe decreases, the length of the thermal-stabilization region increases and the stationary value of Nu_1 is independent of the parallel thermal conductivity.

If the resistance of the near-wall zone is taken into account, the total heat-transfer coefficient is determined as

$$\alpha = \left(\frac{1}{\alpha_{\mathrm{w}}} + \frac{1}{\alpha_1}\right)^{-1}, \tag{44}$$

and, in the case of steady-state heat exchange

$$\alpha^* = \left(\frac{1}{\alpha_w} + \frac{R}{4\lambda_r}\right)^{-1} . \tag{45}$$

The stationary value of the dimensionless heat-exchange coefficient is equal to

$$\mathrm{Nu}_d^* = \left(\frac{1}{10A + 0.061\mathrm{Re}_d\mathrm{Pr}} + \frac{D/d}{8\,(\lambda_s^0/\lambda_f + 0.1\mathrm{Re}_d\mathrm{Pr})}\right)^{-1} . \tag{46}$$

The values of the heat-transfer coefficient α^* determined from (46) are given in Table 1 at D = 0.2 m, d = 0.003 m, A = 1 (glass balls), λ_s = 0.56 W/(m·K), Pr = 0.72 (the air is filtered), T^{in} = 373 K, ρ_f = 0.95 kg/m^3, μ_f = 0.0000184 kg/(m·sec), λ_f = 0.033 W/(m·K), and ε = 0.4. For comparison, we present the heat-transfer coefficient α' for the case of a laminar flow in a tube without a granular layer: Nu_D = 4.36 [9]; in the turbulent flow regime, the dimensionless heat-transfer coefficient is determined from the formula [9]

$$\mathrm{Nu}_D = 0.021\mathrm{Re}_D^{0.8}\mathrm{Pr}^{0.43} .$$

It is seen that in a tube filled with a granular layer, the heat-transfer coefficient is much larger than in a tube through which a pure air flows.

CONCLUSIONS

1. Dependences for calculating the active length of a heat exchanger at the first-kind boundary conditions (formulas (19)) have been obtained.

2. It has been established that the length of the thermal-stabilization region increases with increase in the parallel thermal conductivity (Figs. 3 and 5).

3. Relations for calculating the average dimensionless temperature and the total heat flow under the first-kind boundary conditions (relation (20)) have been obtained.

4. It has been established that the stationary value of the heat transfer coefficient is independent of the parallel thermal conductivity of the granular layer ((28) at the first-kind boundary conditions and (43) at the second-kind boundary conditions).

NOTATION

a_f, thermal diffusivity of a medium, m^2/sec; Bi = KR/λ_r, Biot number; c_f, heat capacity of a medium, W/(kg·K); d, diameter of particles, m; d_e, $4\varepsilon d/6(1-\varepsilon)$, equivalent (hydraulic) diameter of a pore channel, m; D, inner diameter of a heat exchanger, m; J_0 and J_1, first-kind Bessel functions of the zero and first orders; l, thickness of the near-wall zone, m; L, length of the heat exchanger, m; L_a, active length of the heat exchanger, m; $L' = \lambda_r L/(R^2 c_f \rho_f u)$; $L'_a = \lambda_r L_a/(R^2 c_f \rho_f u)$; $\mathrm{Nu} = K_\Sigma R/\lambda_r$, $\mathrm{Nu}_1 = \alpha_1 R/\lambda_r$, $\mathrm{Nu}_d^* = \alpha^* d/\lambda_f$, $\mathrm{Nu}_D = \alpha' D/\lambda_f$, Nusselt numbers; Pe = $Rc_f\rho_f u/(\lambda_x\lambda_r)^{1/2}$, Peclet number; Pr = $\mu_f/(a_f\rho_f)$, Prandtl number; $Q = 2\pi R\int_0^x q dx$, heat flow removed from the region of length x, W; $Q' = Q/Q_\infty$; q, density of a heat flow, W/m^2; r, radial coordinate, m; $r' = r/R$; $R = D/2$; $\mathrm{Re}_d = ud\rho_f/\mu_f$, $\mathrm{Re}_D = uD\rho_f/\mu_f$, $\mathrm{Re}_e = ud_e\rho_f/(\varepsilon\mu_f)$, Reynolds numbers; T, temperature, K; $\langle T\rangle$, temperature averaged over the cross section of the tube x = const, K; T^{in}, temperature of the gas (liquid) at the input of the tube, K; T_0, ambient temperature, K; u, rate of gas (liquid) filtration, m/sec; x, longitudinal coordinate, m; $x' = (\lambda_r x)/(R^2 c_f \rho_f u)$; α_w, near-wall heat-transfer coefficient, W/(m^2·K); α_1, α, α^*, heat-transfer coefficients in (39), (44), (45), W/(m^2·K); δ_m, thickness of the tube wall, m; ε, porosity, $\theta = (T - T_0)/(T^{in} - T_0)$ and $\theta = (T - T^{in})/T^{in}$, dimensionless relative temperatures at the first-kind and second-kind boundary conditions; λ_f, thermal conductivity of the medium, W/(m·K); λ_x and λ_r, parallel and cross-field thermal conductivities of the granular layer, W/(m·K); λ_m, thermal conductivity of the tube material, W/(m·K); λ_s, thermal conductivity of the material of the particles, W/(m·K); λ_s^0, thermal conductivity of the granular layer at $u = 0$, W/(m·K); μ_f, dynamic viscosity of the medium, kg/(m·sec); ρ_f, density of the medium, kg/m^3. Subscripts: a, active; e, equivalent; f, medium (gas or liquid); in, input; m, material of the tube; s, particle; w, near-wall; Σ, total.

REFERENCES

1. M. É. Aérov, O. M. Todes, and D. A. Narinskii, *Apparatus with a Stationary Granular Layer* [in Russian], Khimiya, Leningrad (1979).
2. M. É. Aérov and N. N. Umnik, Coefficients of thermal conductivity in a granular layer, *Zh. Tekh. Fiz.*, **21**, Issue 11, 1351–1363 (1951).
3. V. A. Mukhin and N. N. Smirnova, *Study of Heat and Mass Transfer Processes in the Process of Filtration in Porous Media* [in Russian], Preprint No. 26–78 of the Institute of Thermal Physics of the Siberian Branch of the USSR Academy of Sciences, Novosibirsk (1978).
4. R. A. Dekhtyar', D. F. Sikovskii, A. V. Gorin, and V. A. Mukhin, Heat transfer in a granular layer at moderate Reynolds numbers, *Teplofiz. Vys. Temp.*, **40**, No. 5, 748–755 (2002).
5. Yu. S. Teplistkii, Heat exchange in a tube filled with granular layer, *Inzh.-Fiz. Zh.*, **77**, No. 1, 86–92 (2004).
6. N. I. Gel'perin and V. G. Ainshtein, *Fluidization* [Russian translation], Khimiya, Moscow (1974).
7. V. A. Borodulya and Yu. P. Gupalo, *Mathematical Models of Chemical Reactors with a Fluidized Bed* [in Russian], Nauka i Tekhnika, Minsk (1976).
8. A. V. Luikov, *Heat Conduction Theory* [in Russian], Vysshaya Shkola, Moscow (1967).
9. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, *Heat Transfer* [in Russian], 2nd edn., Énergiya, Moscow (1969).